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3–5 Equations with the Variable on Both Sides

Objective: To solve equations with the variable on both sides.

Example 1 Solve $5x = 2x + 15$.

Solution

$5x - 2x = 2x + 15 - 2x$ Subtract $2x$ from each side.

$3x = 15$

$x = 5$

The solution set is $\{5\}$.

Check: $5(5) \stackrel{?}{=} 2(5) + 15$

$25 \stackrel{?}{=} 10 + 15$

$25 = 25\ \surd$

Example 2 Solve $4x = 30 - x$.

Solution

$4x + x = 30 - x + x$ Add x to each side.

$5x = 30$

$x = 6$ The solution set is $\{6\}$.

Solve.

1. $5n = 3n + 8$ **{4}**
2. $7a = 2a + 30$ **{6}**
3. $y = 20 - 3y$ **{5}**
4. $3b = 80 - 5b$ **{10}**
5. $10n = 36 - 2n$ **{3}**
6. $2x = 20 - 8x$ **{2}**
7. $21a = 56 + 7a$ **{4}**
8. $30 + 6x = 12x$ **{5}**
9. $-9a = -12a - 45$ **{−15}**
10. $33c + 60 = 21c$ **{−5}**
11. $72 - 4n = -22n$ **{−4}**
12. $-11a = -12a - 21$ **{−21}**

Example 3 Solve $\frac{2}{5}x + 12 = x$.

Solution

$\frac{2}{5}x + 12 - \frac{2}{5}x = x - \frac{2}{5}x$ Subtract $\frac{2}{5}x$ from each side.

$12 = \frac{5}{5}x - \frac{2}{5}x$ Rewrite $1x$ as $\frac{5}{5}x$.

$12 = \frac{3}{5}x$

$\frac{5}{3} \cdot \frac{12}{1} = \frac{5}{3}\left(\frac{3}{5}x\right)$ Multiply each side by $\frac{5}{3}$, the reciprocal of $\frac{3}{5}$.

$20 = x$ The solution set is $\{20\}$.

Example 4 Solve $\frac{6 + x}{3} = x$.

Solution

$3\left(\frac{6 + x}{3}\right) = 3 \cdot x$ Multiply each side by 3, the reciprocal of $\frac{1}{3}$.

$6 + x = 3x$

$6 + x - x = 3x - x$ Subtract x from each side.

$6 = 2x$

$3 = x$ The solution set is $\{3\}$.

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3–5 Equations with the Variable on Both Sides *(continued)*

Solve.

13. $\frac{2}{3}x - 5 = x$ **{−15}**
14. $\frac{3}{4}x - 8 = x$ **{−32}**
15. $x = \frac{1}{2}x + 7$ **{14}**
16. $x = \frac{4}{5}x - 9$ **{−45}**
17. $\frac{x - 2}{3} = x$ **{−1}**
18. $\frac{3 + y}{4} = y$ **{1}**
19. $y = \frac{7 - 2y}{5}$ **{1}**
20. $x = \frac{9 + x}{4}$ **{3}**

Vocabulary

Empty set or null set The set with no members.

Identity An equation that is true for every value of the variable(s).

Symbol ϕ (empty set, or the null set)

CAUTION An equation may have no solution, or it may be satisfied by every real number.

Example 5 Solve: **a.** $5(a - 2) - 3 = 3a + 7 + 2a$ **b.** $\frac{1}{3}(24x - 15) = 8x - 5$

Solution

a. $5a - 10 - 3 = 5a + 7$

$5a - 13 = 5a + 7$

$-13 = 7$ ← **False**

The equation has *no solution.*

b. $8x - 5 = 8x - 5$ ← **Identity**

An identity is true for every value of the variable.

The solution set is {real numbers}.

Solve each equation. If the equation is an identity or if it has no solution, write *identity* or *no solution.*

21. $2(x - 3) = 5x$ **{−2}**
22. $4(y - 5) = 9y$ **{−4}**
23. $3n = 6(3 - n)$ **{2}**
24. $-3m = 5(2 - m)$ **{5}**
25. $2(a - 1) = 2a + 3$ **No solution**
26. $\frac{1}{4}(28x - 8) = 7x - 2$ ~~Identity~~
27. $\frac{1}{3}(3x - 3) + 2 = 2x$ **{1}**
28. $4(a - 1) - 5 = 3a + 7$ **{16}**
29. $3(5 + y) - y = 2y + 15$ ~~Identity~~
30. $4a + 7 + a = 3(a - 1)$ **{−5}**
31. $\frac{3n - 15}{4} = 2n$ **{−3}**
32. $\frac{2n - 9}{2} = n$ **No solution**

Mixed Review Exercises

Simplify.

1. $3 + \left(-\frac{1}{3}\right) + \left(-\frac{5}{3}\right)$ **1**
2. $-2\frac{3}{4} + 1\frac{1}{4}$ **$-1\frac{1}{2}$**
3. $-115 - (-10)$ **−105**
4. $15x + (-3x) - 2$ **$12x - 2$**
5. $-4y + 5 + 18y + 23$ **$14y + 28$**
6. $6(-2)(-5)(-4)$ **−240**

Solve.

7. $-2 - x = 5$ **{−7}**
8. $4 + (1 + k) = 2$ **{−3}**
9. $3x = -276$ **{−92}**
10. $\frac{1}{2}x = 3\frac{1}{2}$ **{7}**
11. $\frac{x}{6} = 7$ **{42}**
12. $-10\frac{2}{3} = -\frac{1}{3}x$ **{32}**